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Ellipse-based leg-trajectory generation for galloping quadruped robots †

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Abstract

Controlling the motions of the front and rear legs and regulating the compliance of the legs are important for stable gallop. In this paper, a new method called ellipse-based trajectory generation method (ETGM) to generate foot trajectories for galloping quadrupeds is proposed. Unlike many previous works which attempted controlling foot trajectory, which need a sophisticated algorithm to avoid forcing the feet out of the workspace and thus making galloping unstable, a new trajectory generation method is based on an elliptic trajectory with constant radii but with changes in its center position. The rotational speed of the elliptic trajectory or the orbit trajectory is determined by the desired height of galloping and the running speed. It is assumed that each leg of a galloping quadruped robot has passive ankle joints with passive springs, thus acting as a spring loaded inverted pendulum (SLIP). To check the performance and effectiveness of the proposed method, a series of computer simulations of a 2-D quadruped robot galloping in the sagittal plane were performed. The simulation results show that the proposed method is simple to implement and very effective in generating stable gallop.

Keywords: Quadruped robots; Gallop; Touch-down; Ellipse-based trajectory; Force control

1. Introduction

There are numerous researches being carried out on autonomous field robots. Because of prominent stability issues, many studies have been done about mobile robots with wheels. However, due to their limited mobility in some environments, such as stairs or rough terrains, research and development on humanoids and other legged robots is needed.

Legged robots are, by nature, unstable or prone to falling. Thus, the research on them is concentrated on enhancing their locomotion stability. Static locomotion has been used in many applications of legged robots to guarantee their locomotion stability, which significantly slows down the speed of the robot. Recent research progress on humanoids has made it possible to employ dynamic locomotion, which increases the speed of the robot motion. Most of the legged robots with capability of dynamic and static locomotion control their body posture and locomotion stability by maintaining at least one leg in full contact with the ground. This type of walking still has a limit in the speed of travel, which calls for the need of running and galloping where all the feet are off the ground for some period of time.

Running of bipeds is a successive action with a flight phase, during which the robot is completely off the ground, and a stance phase, during which at least one of the two feet is in contact with the ground. This kind of running motion is very similar to the motion of a ball repeatedly bouncing off the ground. During the flight phase, only the gravitational force is applied at the robot, so its momentum about the CG (center of gravity) is conserved. Thus, the gross motion of the

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robot in the air is determined by its interaction with the ground during the previous stance phase. The contacts in the stance phase are typically modeled with the so-called spring-loaded inverted pendulum model (SLIP) [1-3]. Hopping movements were realized through research on contact conditions of hopping robots [1]. They include the angle between the center of mass and the foot, the stiffness of the contact leg, and the contact velocity. The relations between these parameters and running speed, and flight phase length were researched by Raibert [4]. Furthermore, animal running with energy savings through exchanges of the potential and kinetic energy is explained with the pogo stick theory [5]. How the stability of a 3-D SLIP is influenced by the leg position at a touchdown was analyzed with an explicit approximate stride map [6].

The running of a quadruped with one rotational active and passive linear spring on each leg was attempted by [7]. However, due to the similarity of the legs to the SLIP, they have unnatural structure, which is different from typical legs of mammals, and have restrictions in their motions. A concept based on templates and anchors was proposed by [8]. A template is a simple guide for locomotion control, and an anchor, a more elaborate dynamic system, embeds the behaviors of its templates within it [8]. The so-called central pattern generation (CPG) method is proposed to control the locomotion of quadruped robots. Based on this, walking of a quadruped robot was controlled by the signals from the CPG and reflex sensor feedback [9].

This work proposes a new, simple and effective method to generate leg motions for galloping based on elliptic motions around the shoulder and pelvic joints. This makes it possible to generate various galloping motions simply by controlling parameters such as the center of the trajectory ellipse, major and minor axes as well as orbiting speed. This paper also proposes how the robot interaction with the ground is regulated to realize SLIP motions. For simplicity, these key ideas are verified in a series of computer simulations of a 2D quadruped robot in gallop.

This paper is organized as follows. In Section 2, the quadruped robot model used in this work is introduced and the spring-loaded inverted pendulum (SLIP) concept is reviewed along with the definition of gallop phases. In Section 3, the method to generate a gallop trajectory based on an ellipse is explained. In Section 4, the results of computer simulations are described, followed by conclusions in Section 5.

2 Gallop of quadrupeds

2.1 Quadruped robot model

The quadruped robot currently under development is similar to a large dog in its size when it is built. Its basic structure is shown in Fig. 1. Each leg has two active joints for pitch and roll motions near the body, and one knee joint and a passive ankle joint. The passive ankle joint performs the function of a spring of the SLIP in gallop. A pair of gyro sensors at the center of the body measure the gallop speed. Contact switches are to be installed at the sole of each foot to detect contacts with the ground.

This work is limited to 2-D motions in the sagittal plane. The 2-D model of the robot on the sagittal plane is shown in Fig. 2 with the orientations of the front and rear legs. The parameters on the joints and the links are summarized in Table 1. The dynamics of the 2-D quadruped robot is simulated in this work.

2.2 Gallop phases

A single cycle of gallop by animals can be largely divided into two phases: flight phase and stance phase [10]. The posture and the speed at a touchdown are the most important factors in affecting the gallop motion. For more detailed analysis, we propose that the flight phase and the stance phase are further divided into two phases respectively, resulting in four phases as shown in Fig. 3. Thus, the flight phase consists of Phase IV and Phase I, and the stance phase

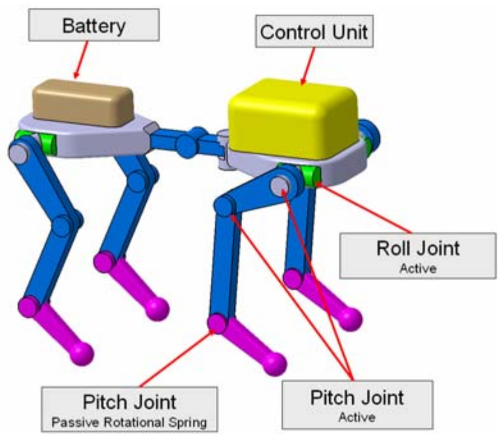
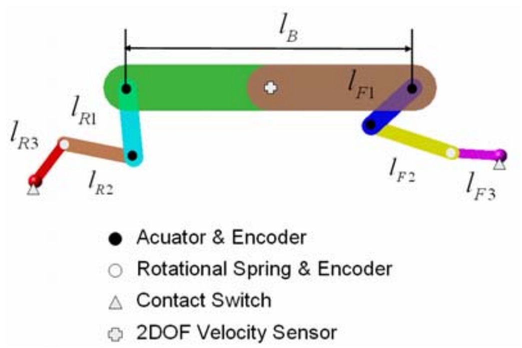


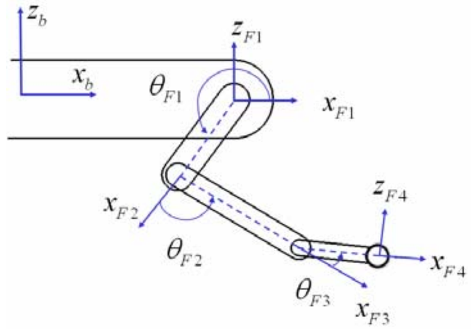
Fig. 1. Configuration of a quadruped robot considered in the work.

Table 1. Parameters of the quadruped robot (Symbols F, R, U, and L denotes front, rear, upper and lower, respectively.).

Parameters	Values
FU leg	120 mm, 0.475 kg
FL leg	180 mm, 0.479 kg
RU leg	150 mm, 0.569 kg
RL leg	150 mm, 0.306 kg
Body	600 mm, 17.818 kg
Ankle Stiffness	80.0 N-m/rad
Ankle Damping	1.0 N-m-s/rad



(a) 2D Dynamic model



(b) Description of orientation

Fig. 2. Sensors and kinematics in the 2-D model.

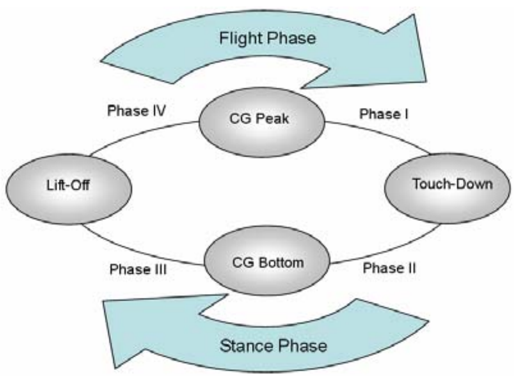


Fig. 3. Galloping phases.

Table 2. Gallop phases.

Phases	Conditions	Foot Contact Sensor
I	$\dot{z}_{CG} < 0, \ddot{z}_{CG} = -g$	OFF
II	$\dot{z}_{CG} < 0$	ON
III	$\dot{z}_{CG} > 0$	ON
IV	$\dot{z}_{CG} > 0, \ddot{z}_{CG} = -g$	OFF

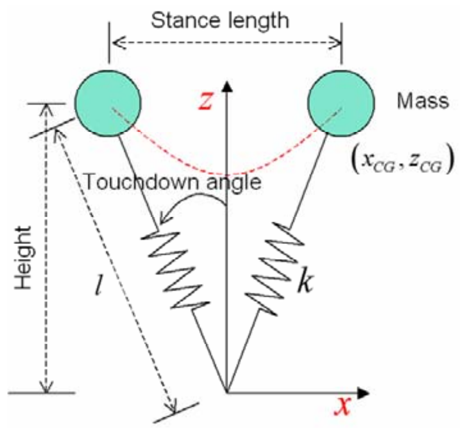


Fig. 4. Spring-loaded inverted pendulum (SLIP) model.

consists of Phase II and III. The end of Phase IV and the start of Phase I is called 'CG top,' when the center of gravity of the robot becomes at its highest position. The end of Phase II and the start of Phase III is called 'CG bottom,' when the center of gravity of the robot becomes at its lowest position. 'Touch-down' is the moment a foot comes in contact with the ground and the beginning of Phase II, or the beginning of the stance phase. The start of Phase IV is 'lift-off,' with all the foot leaving from the ground and the body moving upward. Table 2 shows mathematical conditions and status of touch sensors at the foot soles in each phase.

2.3 SLIP model

In many previous researches on the SLIP, a typical model of which is shown in Fig. 4, the three most important factors of motion are identified: touchdown angle, stiffness of the leg, and the speed in the motion direction at contacts with the ground. Touchdown angle affects the forward motion speed and the height of the flight. As it decreases, the forward speed increases and the height of the flight decreases. The stiffness of the legs enables the robot to store its kinetic energy in the form of potential energy, which could be used in take-off phases. It also affects the period of a stance phase, which is determined by the natural frequency of a mass-spring system, where the mass and the spring represent the body of the quadruped and the spring coefficients at the feet, respectively. The speed in the direction of movement at contacts with the ground is simply needed for the SLIP model to advance continuously.

3. Foot trajectory generation

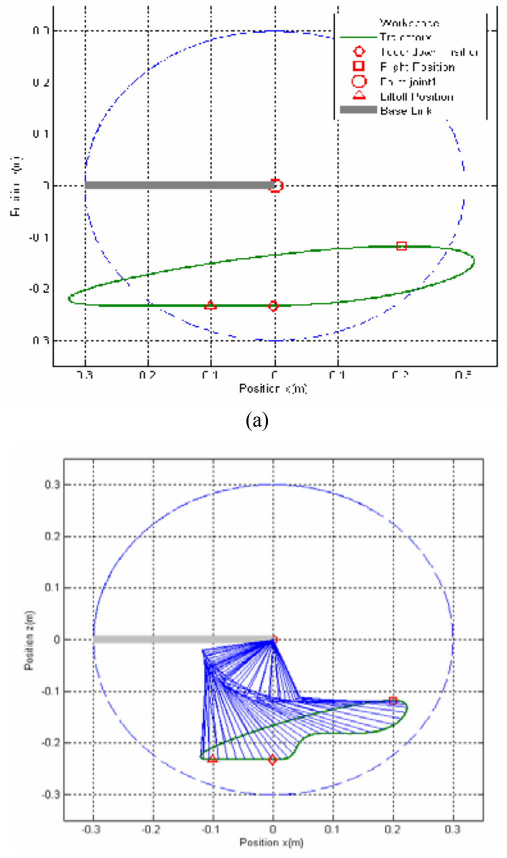
Along with the stiffness of the leg, the trajectory of the foot is a very important factor that influences the stability of gallop. Good foot adaption to the ground between the touch-down and the lift-off, storing kinetic energy of the robot during Phase II, and re-utilizing the stored energy effectively during Phase III are critical to saving energy and maintaining gallop stability.

3.1 Conventional methods

To accommodate the ground profile effectively, the foot trajectory may be specified relative to the ground level. The trajectory may have a few via-points depending on the walking pattern, each of which a robot foot has to pass through each of the via-points with a designated velocity. And then a polynomial interpolation could be used to connect them to form a smooth leg trajectory. Fig. 5 shows an example of foot trajectory formed with a cubic polynomial interpolation of touch-down, lift-off, and CG top point with passing speeds. As shown in the figure, the resulting foot trajectory gets out of the workspace of the foot. To resolve this problem, some portions of the trajectory may be modified with different interpolation schemes; however, the resulting trajectory within the workspace may be highly fluctuant, as can be seen in Fig. 5.

3.2 Ellipse-based trajectory generation method (ETGM)

To solve issues on specifying foot trajectory within their workspace with conventional methods and to simplify the scheme to design foot trajectory, we suggest the ellipse-based trajectory generation method (ETGM). The proposed ETGM is based on an ellipse with a fixed size but with its center position variable relative to the pivot points of the legs. Fig. 6 shows an ellipse. Its center position, (x_c, z_c), is to be changed appropriately. Its orbit angle, $\phi(t)$, and its speed, $\dot{\phi}(t)$, are to be changed appropriately. Point O is the position of the shoulder or the pivot point of leg rotations, which is fixed at the body of the quadruped robot along with the coordinate frame $\{OXZ\}$. Since only the center position of the trajectory ellipse with a fixed size is changeable, it is easy to avoid the



(a)

(b)

Fig. 5. Foot trajectories using conventional methods: (a) the trajectory getting out of the workspace, and (b) the trajectory staying within the workspace but only with many computations due to many small sections for interpolation.

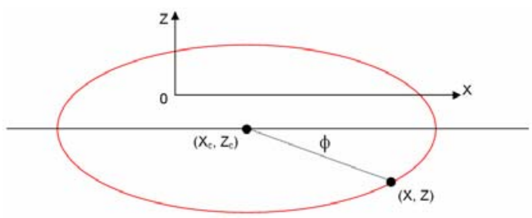


Fig. 6. Ellipse for leg motions.

situation where the desired foot position goes out of the reach of the leg.

If the foot trajectory is based on an ellipse represented by

$$\left(\frac{x-x_c}{\rho_1}\right)^2+\left(\frac{z-z_c}{\rho_2}\right)^2=L^2$$

where ρ_1 and ρ_2 are the half of the length of the major and minor axes respectively, and

$$x(t)=\rho_1\cos(\omega t)+x_c \tag{1a}$$

$$z(t)=\rho_2\sin(\omega t)+z_c\ . \tag{1b}$$

Since the distance between the orbiting point (*x, z*) and point *O* should be less than the length of the corresponding leg,

$$x(t)^2+z(t)^2\le L^2 \tag{2}$$

where *L* is the length of the fully stretched swing leg. Angular trajectory of $\phi(t)$ is to be determined by polynomial interpolations with the angular positions and the angular velocities of the foot at the touch-down, lift-off and CG-top. Resulting $\phi(t)$ is a function of time.

3.3 Changes in other ellipse parameters

Note that depending on the speed of travel, the length of the major and minor axes of the ellipse $2\rho_1$ and $2\rho_2$ could be also changed. It is also feasible to change the orientation of the ellipse relative to the coordinate {*OXY*}, even though its advantages over the ETGM with only changes in the ellipse center are not distinct. The ETGM with *simultaneous* variations of all the parameters is feasible, but may pose some control problems due to over-specifications of a foot trajectory.

3.4 Orbit trajectory

Once the trajectory ellipse is fixed, its orbit trajectory, $\phi(t)$, has to be determined. There are many possible ways to select $\phi(t)$. Here, one example of specifying $\phi(t)$ is introduced, which will be used in simulations in the following section.

Suppose the orbit angles when the quadruped robot is at its CG-top, touch-down, and lift-off are determined. Cubic polynomials in time are used for interpolation for the orbit trajectory in Phase IV and Phase I. For a steady push-back of the foot on the ground, a constant orbit trajectory is used in Phase II and III.

Thus,

$$\phi_1(t)=a_{13}t^3+a_{12}t^2+a_{11}t+a_{10}\quad\text{(Phase I)} \tag{3}$$

$$\phi_2(t)=a_{23}t^3+a_{22}t^2+a_{21}t+a_{20}\quad\text{(Phase IV)} \tag{4}$$

$$\phi_3(t)=a_{31}t+a_{30}\quad\text{(Phase II \& III combined)} \tag{5}$$

There are 10 constants to be determined.

For continuation of the orbit trajectory and the orbit speed,

$$\phi_1(T_{pk})=\phi_2(T_{pk}) \tag{6}$$

$$\phi_2(T_{td})=\phi_3(T_{td}) \tag{7}$$

$$\phi_3(T_{lo})=\phi_1(T_{lo}) \tag{8}$$

$$\dot\phi_1(T_{pk})=\dot\phi_2(T_{pk}) \tag{9}$$

$$\dot\phi_2(T_{td})=\dot\phi_3(T_{td}) \tag{10}$$

$$\dot\phi_3(T_{lo})=\dot\phi_1(T_{lo}) \tag{11}$$

where T_{pk}, T_{td}, and T_{lo} are the time of CG-top, touch-down, and lift-off, respectively.

Suppose $\phi(T_{pk})=\phi_{pk}^*$, $\phi(T_{td})=\phi_{td}^*$, and $\phi(T_{lo})=\phi_{lo}^*$ are given or determined by some method. Then,

$$\phi_1(T_{pk})=\phi_{pk}^* \tag{12}$$

$$\phi_2(T_{td})=\phi_{td}^* \tag{13}$$

$$\phi_3(T_{lo})=\phi_{lo}^* \tag{14}$$

In addition, if the linear speed of the foot on the ground along the ground is v_0, an additional constraint can be obtained by an approximation:

$$\dot\phi_3=a_{31}=v_0/\rho^* \tag{15}$$

where

$$\rho^*=\sqrt{\rho_1\rho_2}\ \text{ or }\ \rho^*=\rho_2\,.$$

From Eq. (6) to Eq. (15), there exist total 10 constraints. Thus, the orbit trajectory that consists of ϕ_1, ϕ_2, and ϕ_3, which has total 10 unknowns, is uniquely determined.

In actual implementations, the lift-off time, T_{lo}, cannot be predetermined before the actual lift-off occurs since it depends on the ground conditions and the strategy to control the robot body. Thus, *Tlo* will be the time of the moment when the all the contact sensors are OFF. Since *Tlo* no longer imposes a constraint in determining the orbit trajectory, an additional constraint can be given. One example is the constraint on the orbiting speed at CG-top, i.e., $\dot{\phi}(T_{pk})$.

4. Simulations

To verify the effectiveness of the ETGM, the motion of a quadruped galloping robot in the sagittal plane is simulated. The orbit of a single ellipse is used for both front and rear legs. Orbit trajectory is obtained based on fixed angles of CG-top, touch-down, and lift-off. The speed of galloping, v_0, is 2.5 m/s. The parameters of the foot trajectory for the quadruped robot are shown in Table 3. As explained in Section 3.4, instead of using a fixed *Tlo*, constraint

$$\dot{\phi}(T_{pk}) = \frac{v_0}{\rho^*}$$

is used in solving for the orbit trajectory for the front and rear feet.

In the simulation, force feedback control is implemented to maintain a constant gallop height. The gallop height is about 310 mm. Since this is not directly relevant to the current work, detailed descriptions on it are not given here. By controlling the elevation of the trajectory ellipse, z_c in Eq. (1), ground reaction force is regulated.

The dynamics of the quadruped robot is simulated with a commercial software called RecurDyn®. The normal and tangential reaction forces from the ground are simulated. Force normal to the ground, f_n, is generated by

Table 3. Parameters of foot trajectories.

Parameters	Font Foot	Rear Foot
x_c	-50 mm	-20 mm
z_c	-160 mm	-145 mm
ρ_1	100 mm	100 mm
ρ_2	30 mm	30 mm
ϕ^*_{pk}	10°	120°
ϕ^*_{td}	-83°	-32°

$$f_n = k\delta^{1.3} + c\dot{\delta}\delta^2 \tag{16}$$

where δ is the combined deformation at the robot foot and the ground, $k = 1.0\times10^6$, and $c = 1.0\times10^3$. Tangential force to the ground, i.e., frictional force, f_h is generated by

$$f_h = \mu(\dot{\delta})|f_n| \tag{17}$$

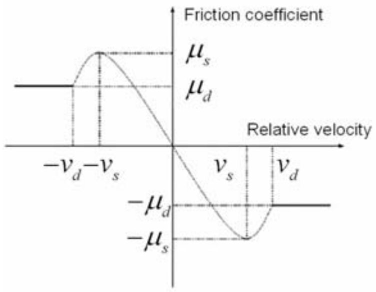


Fig. 7. Friction model for stick-slip.

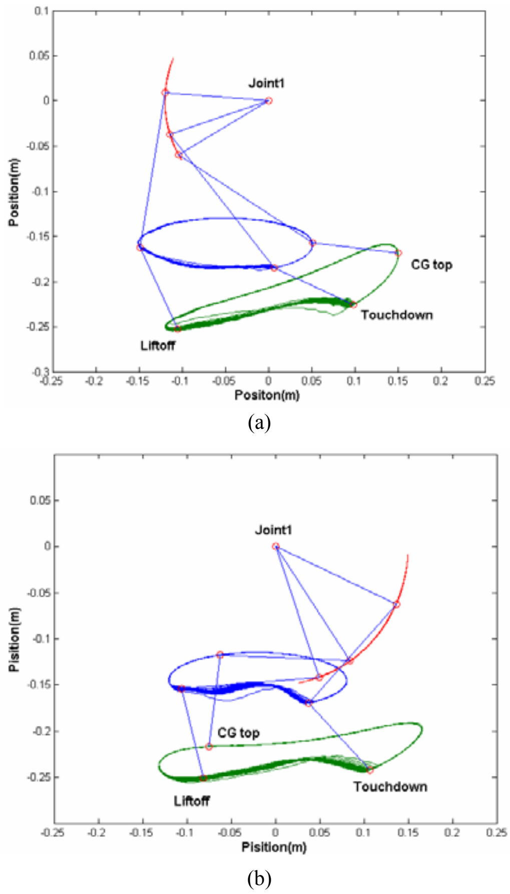
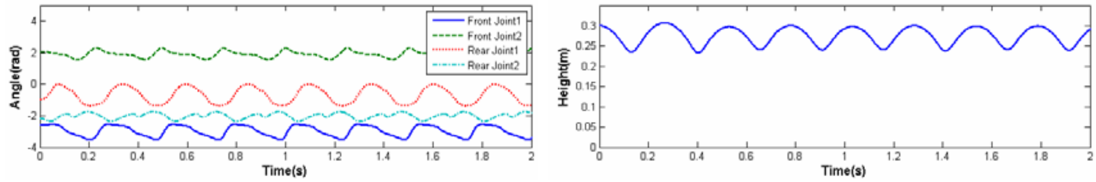
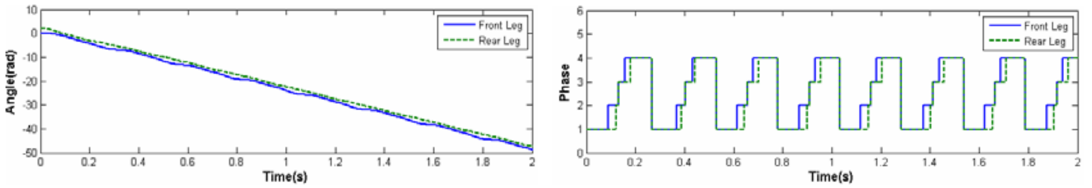


Fig. 8. Traces of (a) the front foot and (b) the rear foot.



(a) (b)



(a) (b)

Fig. 9. Stable gallop is verified in the plots of (a) angular trajectory of the joints, (b) trajectory of the center of gravity, (c) phase transitions, and (d) trajectory of the orbit angles.

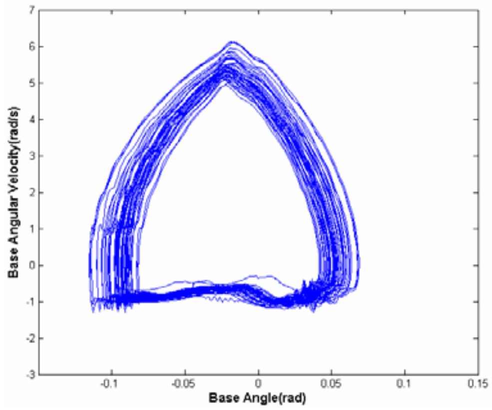


Fig. 10. Phase plot of the body orientation shows a limit cycle.

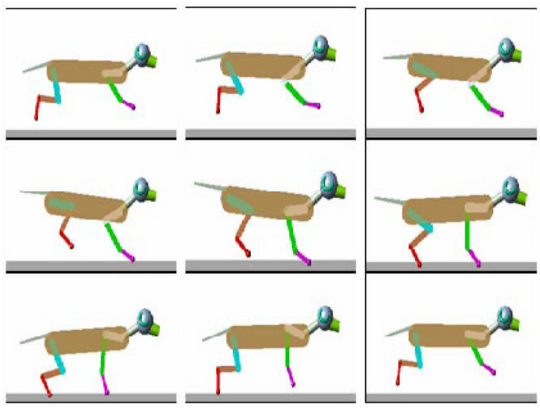


Fig. 11. Consecutive snap shots of a graphic simulation of a galloping quadruped.

where $\mu(\cdot)$ is a friction coefficient function shown in Fig. 7. Friction parameters of $\mu_s = 1$, $\mu_d = 0.8$, $v_s = 0.010$ m/s and $v_d = 0.015$ m/s are used in simulation.

A single cycle of the resulting gallop motion of the quadruped robot is shown in Fig. 8 in the form of a stick diagram. This Fig. shows that the quadruped robot successfully gallops and that each foot tracks the desired foot trajectory generated by the proposed ETGM. The trajectories of the tips of the feet are also elliptic despite the fact that the angles at the passive ankles, each of which has a spring and a damper, are changing due to the ground reaction force. It is interesting to observe that the major axis of elliptic trajectory of the front foot is not parallel to the ground while that of the rear foot is.

The height of the body of the quadruped robot and the orbit angles of the font and rear foot are shown in Fig. 9, which confirms successfully repeating gallop motion. Joint angular positions, the height of the center of gravity of the body, and orbit angular trajectory as well as phase transitions are shown in Fig. 9. It can be seen that the front foot contacts the ground sooner than the rear foot to start Phase II. The phase plot of the body orientation in Fig. 10 shows that robot motions converge to a limit cycle, which indicates stable gallop. Fig. 11 is a series of snap shots of the simulation scene of a galloping quadruped robot.

5. Conclusions

This paper proposes a new method, called elliptic trajectory generation method (ETGM), to generate a foot trajectory for quadruped robots based on trajectory ellipse. Its advantages over other methods include the separation of gross motions of the foot and implementing the SLIP characteristics of the foot, and easy checking of the foot going out of its workspace. The parameters of the trajectory ellipse may be changed depending on gallop speed and gallop height as well as the environment of the gallop. The proposed ETGM was successfully applied to a simple two-dimensional quadruped robot model. Active control of the center position of the trajectory ellipse will be further explored.

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